Can GDP measurement be further improved?
Data revision and reconciliation

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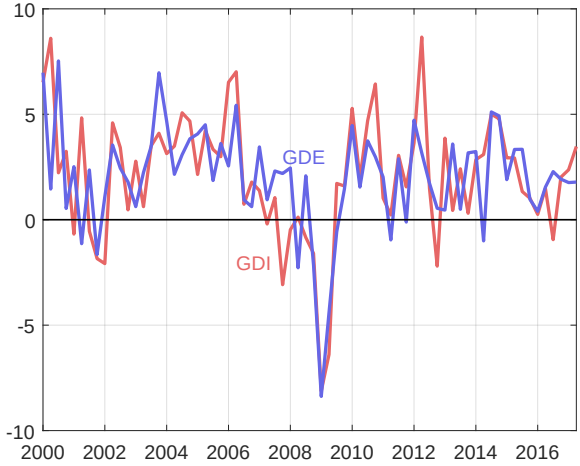
Motivation I

Which is the better measure of GDP? Expenditure (GDE) or Income (GDI)?



Motivation II

Which is the better measure of GDP? Expenditure (GDE) or Income (GDI)?

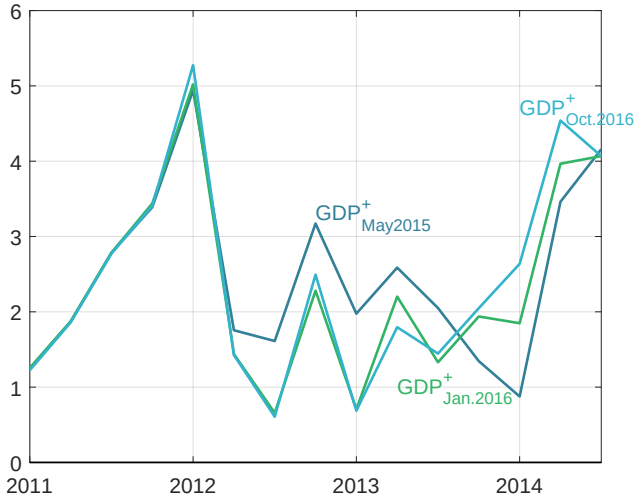
- Nalewaik (2012)
- Chang and Li (2015)

Reconciliation:

- Stone, Champernowne and Meade (1942)
- Weale (1992)
- Diebold (2010)
- Aruoba et al (2013, 2016)
  - FRB Philadephia publishes GDP^{+}
- BEA publishes average ($GDP^{50/50}$)

Motivation III

Following Aruoba et al. (2016), FRB Philadelphia publish GDP$^+$



Problem

Reconciliation relies on assumptions about the errors in the series being reconciled.

- which is more precise?
- lead/lag relationships?
- News or Noise?
  - Is variability due to measurement error?
  - Or does it reflect useful information?

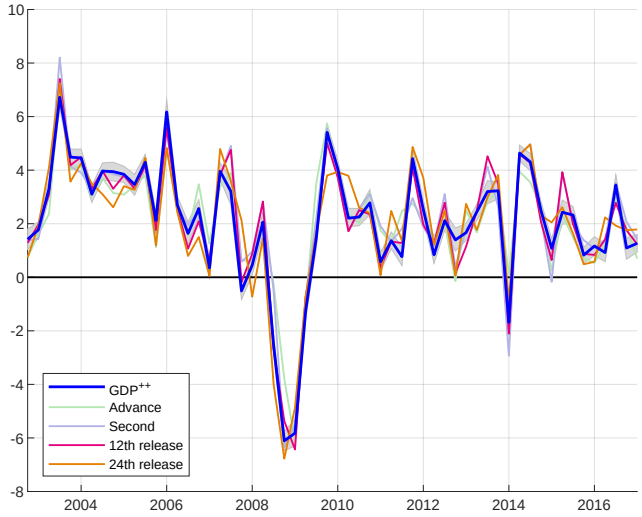
These relationships vary depending on which release(s) we consider.

- Important for producing efficient estimates.
- Important for understanding reliability of estimates.

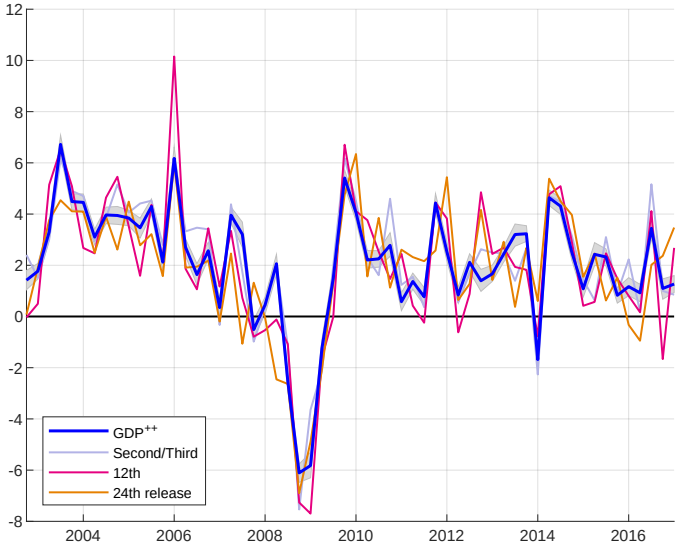
Our Contribution

1. We model the reconciliation problem in a linear state-space framework for data revision (cf Jacobs and van Norden *JEconometrics* 2011)
2. We show how to allow for
   - multiple data releases with varying precision
   - series dynamics
   - news and noise errors, possibly correlated across the two series
3. We show how use of multiple vintages can provide identification.
4. Compare our new measure (GDP^{++}) to real GDE and GDI growth
5. Decompose initial estimates of GDE and GDI growth into news and noise shocks

GDP^{++} vs GDE


10
8
6
4
2
0
-2
-4
-6
-8
2004
2006
2008
2010
2012
2014
2016
GDP++
Advance
Second
12th release
24th release

GDP^{++} vs GDI


12
10
8
6
4
2
0
-2
-4
-6
-8
2004
2006
2008
2010
2012
2014
2016
GDP++
Second/Third
12th
24th release

Revision properties

News and Noise

Let y_t^i be the i-th release of y in period t and $\tilde{y}_t \equiv$ 'true' value of y_t

1. **Noise:**

$$y_t^i = \tilde{y}_t + \zeta_t^i, \qquad \text{cov}(\tilde{y}_t, \zeta_t^i) = 0 \quad \forall i$$

   $\Rightarrow$ revisions (partly) forecastable
   $\Rightarrow$ vintages **more** volatile than 'true' values

2. **News:**

$$\tilde{y}_t = y_t^i + \nu_t^i, \qquad \text{cov}(y_t^i, \nu_t^i) = 0 \quad \forall i$$

   Linked to rational forecasts (De Jong 1987)
   rational statistical agency (Sargent 1989)
   $\Rightarrow$ revisions *cannot* be forecast
   $\Rightarrow$ vintages **less** volatile than "true" values

Notation

GDP_t	real GDP growth ("Truth")
GDE_t	real GDP growth (Expenditure measure)
GDI_t	real GDP growth (Income measure)
GDE_t^i	superscript ^{i} indicates release
GDP_t^+	real GDP growth - FRB Philadelphia measure (after Aruoba et al. 2016)
GDP_t^{++}	our real GDP growth measure
ν_t	News measurement error
ζ_t	Noise measurement error

State-Space Model I

Measurement Equation:

$$\begin{bmatrix} GDE_t^L \\ GDI_t^L \end{bmatrix} = GDP_t + \begin{bmatrix} \nu_{Et}^L \\ \nu_{It}^L \end{bmatrix} + \begin{bmatrix} \zeta_{Et}^L \\ \zeta_{It}^L \end{bmatrix} \tag{1}$$

$$\textbf{Data} = \textbf{Truth} + \textbf{News} + \textbf{Noise}$$

where

GDP_t is a latent variable

$$GDE_t^L = [GDE_t^1, \ldots, GDE_t^l]', \qquad GDI_t^L = [GDI_t^1, \ldots, GDI_t^l]',$$
$$\nu_{Et}^L = [\nu_{Et}^1, \ldots, \nu_{Et}^l]', \qquad \nu_{It}^L = [\nu_{It}^1, \ldots..., \nu_{It}^l]'$$
$$\zeta_{Et}^L = [\zeta_{Et}^1, ..., \zeta_{Et}^l]', \qquad \zeta_{It}^L = [\zeta_{It}^1, ..., \zeta_{It}^l]',$$

News: $E[\nu_{E,t}^j | GDE_t^k] = 0 = E[\nu_{I,t}^j | GDI_t^k] \quad \forall j > k$

Noise: $E[\zeta_{i,t}^j | GDP_t] = 0 \quad \forall i = \{E, I\}\ j = 1, \ldots, L$

State-Space Model II

Transition Equation:

$$\boldsymbol{\alpha}_t = \boldsymbol{T} \cdot \boldsymbol{\alpha}_{t-1} + \boldsymbol{R} \cdot \boldsymbol{\eta}_t \tag{2}$$

where $\boldsymbol{\alpha}_t = [GDP_t, \nu_{Et}^{L\,\prime}, \nu_{It}^{L\prime}, \zeta_{Et}^{L\,\prime}, \zeta_{It}^{L\prime}]'$

Identification results from restrictions on the $\boldsymbol{T}$ and $\boldsymbol{R}$ matrices.

- $\boldsymbol{T}$ has one non-zero element ρ to capture persistence in GDP_t, and

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_1 & \boldsymbol{R}_2 + \boldsymbol{R}_3 & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{V}_l \cdot \mathrm{diag}(\boldsymbol{R}_1) & -\boldsymbol{V}_l \cdot \mathrm{diag}(\boldsymbol{R}_3) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{V}_l \cdot \mathrm{diag}(\boldsymbol{R}_2) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{R}_4 & \boldsymbol{R}_6 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{R}_5 \end{bmatrix} \tag{3}$$

where $R_1, \ldots, R_6$ are $l \times 1$ vectors of σ's
and V_l is an $l \times l$ matix with 1's above the diagonal.

Identification

In a model with l releases of each series

- we have $l \cdot (2l + 3)$ moments
- to estimate $1 + 6l$ parameters (ρ and $\boldsymbol{R}_1, \ldots, \boldsymbol{R}_6$)

Releases	Moments	Parameters	Identified?
$l = 1$	5	7	No
$l = 2$	14	13	Yes
$l = 3$	27	19	Yes
$l = 4$	44	25	Yes

Formal proof follows Komunjer and Ng (2011).

Restrictions on $\boldsymbol{T}$ imply that all serial persistence comes through GDP_t.

- News shocks are part of GDP_t, and so have a persistent effect.
- The behavior of News across releases is tightly restricted (via $\boldsymbol{V}_l$.)

Data and estimation

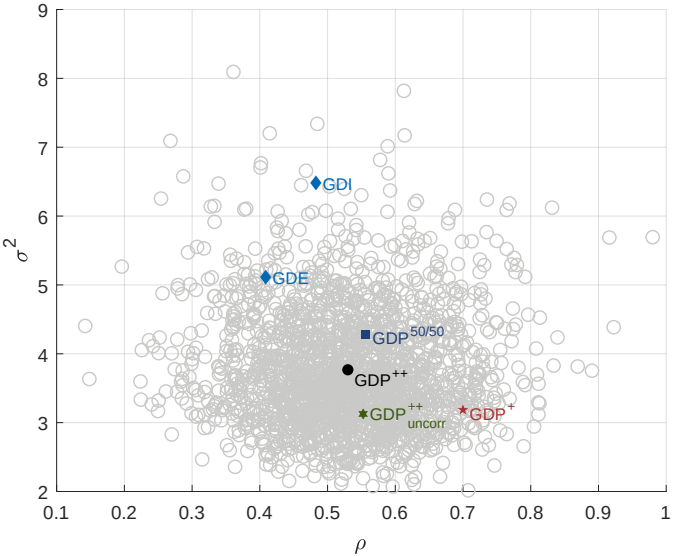
Data

- Monthly vintages of quarterly series 2002Q4–2017Q1
  (earliest available first releases for GDI)
- For real GDE growth
  we use the advance, third, the 12th and the 24th releases
- For real GDI growth
  we use the second/third, the 12th and the 24th releases

Estimation

- Gibbs Sampling with diffuse priors
- Estimate with and without shocks correlated across GDI & GDE

Real GDP growth dynamics



Contribution to GDP_t^{++}

- Use Kalman gain to assess importance of GDP_I and GDP_E at different releases

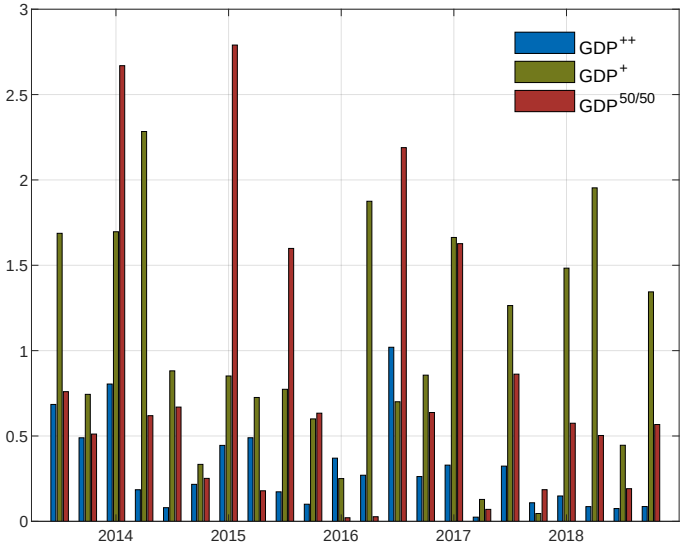
Table: Kalman Gains

	Balanced Sample		Ragged-Edge Sample	
Weight on	GDE	GDI	GDE	GDI
News and Noise				
Advance	0.0272		0.2311	
Second/Third	-0.2103	0.3067	0.3363	0.4804
12th	0.7104	0.1081	0	0
24th Release	0.0479	0.0125	0	0

Comparing GDP Estimates

	GDP^{++}	GDP^{+}	$GDP^{50/50}$	$GDE_{advance}$	GDE_{third}	GDE_{12th}	GDE_{24th}	GDI_{second}	GDI_{12th}	GDI_{24th}
GDP^{++}	1									
GDP^{+}	0.72	1								
$GDP^{50/50}$	0.92	0.83	1							
$GDE_{advance}$	0.93	0.63	0.85	1						
GDE_{third}	0.95	0.59	0.85	0.94	1					
GDE_{12th}	0.97	0.64	0.87	0.93	0.97	1				
GDE_{24th}	0.93	0.71	0.94	0.88	0.89	0.93	1			
GDI_{second}	0.94	0.7	0.85	0.85	0.87	0.85	0.83	1		
GDI_{12th}	0.89	0.75	0.86	0.8	0.81	0.8	0.79	0.87	1	
GDI_{24th}	0.81	0.86	0.95	0.73	0.72	0.73	0.79	0.79	0.84	1

Comparing GDP Revisions



Conclusion

We show how to reconcile series subject to revision due to news and noise.

- Identification possible due to differing impact of news and noise errors across data vintages, and persistence in "true" GDP

We provided a new real GDP growth measure using multiple data vintages. Compared to GDP^{+}, our estimate

- tracks expenditure-based estimates more closely.
- has smaller revisions after initial estimates.

Other potential applications

- 3-way reconciliation with production-based estimates.
- Balance of Payments reconciliation.