

# Modeling Multivariate Data Revisions

Jan P.A.M. Jacobs\*      Samad Sarferaz†  
Jan-Egbert Sturm‡      Simon van Norden§

This version: 28th August 2015

## Abstract

Data revisions in macroeconomic time series are typically studied in isolation ignoring the joint behaviour of revisions across different series. We show how to model such systems with standard linear state space models with measurement errors that can be news or noise. We consider systems of variables, which may be linked by one or more identities and where true values and revisions (or measurement errors) can be correlated. This approach allows for (i) the possibility that early releases of some series may help predict revisions in other series and (ii) relationships statistical agencies may create in producing estimates consistent with accounting identities. We motivate and illustrate our multivariate modeling framework with Swiss current account data.

*JEL classification:* C22, C53, C82

*Keywords:* data revisions, state space form, linear constraints, correlated measurement errors, Bayesian econometrics, current account statistics, Switzerland

---

\*University of Groningen, University of Tasmania, CAMA and CIRANO.

†Corresponding author: KOF Swiss Economic Institute, ETH Zurich, Switzerland  
Leonhardstrasse 21, 8092 Zurich, Switzerland. Tel.: +41 44 632 54 32; Fax: +41 44 632 11 50;  
Email: sarferaz@kof.ethz.ch.

‡KOF Swiss Economic Institute, ETH Zurich, Switzerland and CESifo, Germany

§HEC Montréal, CAMA, CIRANO and CIREQ

# 1 Introduction

Data revision has become an increasingly important field of macroeconomic research in recent years, spurred in part by the creation of major databases of original release data for the US and other major OECD economies. Croushore (2011) provides a survey of the modern literature. Data revisions in individual time series are typically studied in isolation ignoring information in other related series. Zadrozny (2008), Jacobs and van Norden (2011) and Cunningham et al. (2012) are recent examples of univariate data revision models.

This paper extends the Jacobs and van Norden (2011; henceforth JvN) modeling approach to systems of variables allowing for true values and news and noise measurement that cover all sorts of special cases, including

- the distinction of measurement errors into news, i.e. revisions are not forecastable, and noise, i.e. revisions are forecastable;
- new information causing simultaneous revisions in several variables;
- true values and news and noise measurement errors to be correlated across variables;
- true variables of different variables to be linked via accounting identities / linear constraints.
- several variables to have underlying factors for each variable (as in the Illustration shown in this paper), or a single underlying factor (as in e.g. the data reconciliation literature initiated by Stone, Champenowne and Meade (1942)).<sup>1</sup>

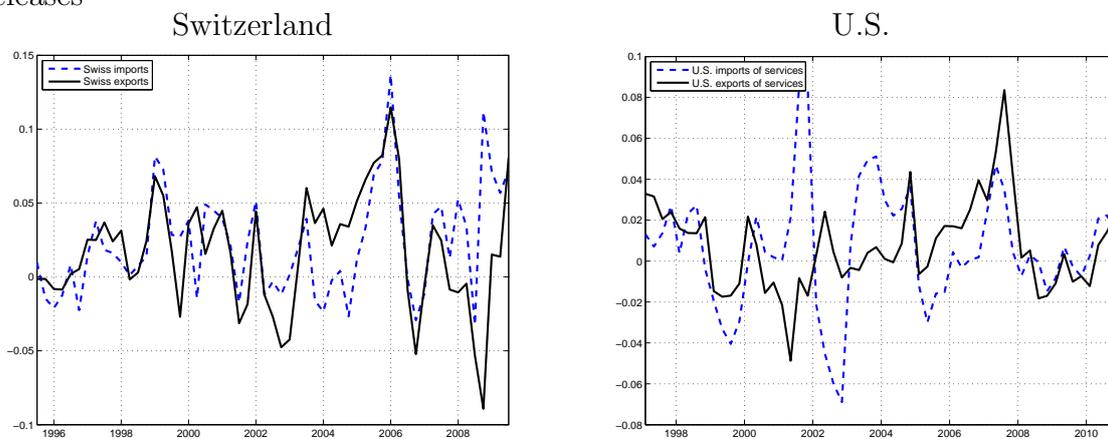
Our framework deviates from the multivariate approaches of Patterson (2003), who models the most recent observations from vintages rather than different vintage

---

<sup>1</sup>Recent contributions to this area include Fixler and Nalewaik (2009) and Aruoba et al. (2012, 2013).

estimates for an observation, and Clements and Galvão (2012), who consider vintage-based VAR models that cannot deal with news or noise (Hecq and Jacobs 2009). Closest to our study is Kishor and Koenig (2012), who use a completely different approach. We further develop the existing literature on modeling multivariate data revisions by allowing for adding-up constraints and explicitly taking into account the possibility that innovations to true values, news and noise can be correlated.

Figure 1: Revision of imports and exports after one year as percentage of first releases



We illustrate our multivariate data revisions modeling approach with a real-time current account data set on Switzerland. Correlations between revisions in imports and exports are higher in Switzerland than in the U.S., the country with the longest span of real-time data, as shown in Figure 1. In addition, distinction of different types of data revisions requires careful handling of the real-time data and in many cases direct access of the officials of the statistical agency. This holds especially for the identification of so-called historical revisions which affect whole vintages of data. By using Swiss data we can build upon the expertise gained in Jacobs and Sturm (2008) for the treatment of historical revisions.

We estimate the parameters of our state-space framework using econometric methods, similar to those of Aruoba et al. (2012, 2013), Carriero, Clements and Galvao (2014) and Schorfheide and Song (2014). We find that multivariate models

of true values with cross-correlated news and noise shocks fit the data better than univariate models of exports and imports.

The paper is structured as follows. Section 2 briefly discusses data revisions and their properties. Section 3 presents our general model, while Section 4 describes estimation methods. Section 5 illustrates our framework with Swiss current account data. Section 6 concludes.

## 2 Data revisions

Real-time data are typically displayed in the form of a data trapezoid. We move to later vintages as we move across columns from left to right and we move to later points in time when we move down the rows. Note that the frequency of vintages need not necessarily correspond to the unit of observation; for example, in our Illustration below the statistical agency publishes monthly vintages of quarterly observations. In this paper it is more convenient to work with releases, i.e. diagonals of the data trapezoid. Therefore we use the following notation: superscripts refer to releases, while subscripts refer to periods. Hence,  $z_t^1$  is the first release for variable  $z$  in period  $t$ .

Three types of data revisions can be distinguished:

1. initial revisions, i.e. changes in the first few observations,
2. annual (seasonal) revisions due to updated seasonal factors and the confrontation of quarterly with annual information, and
3. historical, comprehensive or benchmark revisions, related to changes in e.g. statistical methodology.

The distinction of revisions into these types requires careful handling of the real-time data and in many cases direct access to the officials of the statistical agency. Initial and seasonal revisions are regular and recurring, i.e. can in principle be modeled

and forecast. Historical revisions are much more difficult to handle. Redefinitions like changes of base years do not cause many difficulties; however, methodological changes are much more difficult to deal with. Our model is intended for regular, initial and annual revisions; in our Illustration below we adjust for comprehensive revisions. For details see Section 5.1 below.

Many official statistics are jointly produced by statistical agencies. Therefore true values and news and noise measurement errors may be correlated across variables. Identities linking variables will also cause measurement errors to be correlated across them. To deal with revisions in more than one variable, a multivariate model is required, to which we turn to.

### 3 Methodology

Our multivariate state-space model follows Durbin and Koopman (2001):

$$\text{measurement equation} \quad \mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t \quad (1)$$

$$\text{transition equation} \quad \boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \mathbf{R}\boldsymbol{\eta}_t. \quad (2)$$

In the JvN framework the data vector  $\mathbf{y}_t$  consists of  $l$  different releases  $y_t^i$ ,  $i = 1, \dots, l$  for observation  $t$ . To analyse data revisions in an  $N$ -dimensional data vector, we stack  $l$  releases of the  $N$  variables (first by release then by variable) in the  $Nl \times 1$  data vector  $\mathbf{y}_t$ . So, the dimensions of the variables and error terms in the state-space form are as follows:  $\mathbf{y}_t$  is  $Nl \times 1$ ,  $\boldsymbol{\alpha}_t$  is  $Nm \times 1$ ,  $\boldsymbol{\eta}_t$  is  $Nr \times 1$ . Without loss of generality we include the measurement error  $\boldsymbol{\varepsilon}_t$  in the state vector and assume  $\boldsymbol{\eta}_t \sim N(\mathbf{0}, \mathbf{I}_{Nm})$ .

We begin with a simple state-space structure that ignores measurement errors and then show how successive features may be added to the model. Readers already quite familiar with the univariate data revision model of JvN may simply note that

we stack those univariate models into a multivariate framework and skip ahead to the discussion of data reconciliation and factor structures, and correlated shocks.

## True values

To help fix notation, we begin by ignoring data revision and focusing on the dynamics of our time series. In this simple case,  $l = 1$  so we simply have a single observed series of observations on each of our  $N$  variables. In that case,  $\mathbf{y}_t$  is  $N \times 1$ ;  $\boldsymbol{\alpha}_t = [\mathbf{y}'_t, \boldsymbol{\phi}'_t]'$ , which is  $(N + Nb) \times 1$ , so that  $\boldsymbol{\phi}_t$  is  $Nb \times 1$ ; and  $\mathbf{Z} = \begin{bmatrix} \mathbf{I}_N & \mathbf{0}_{N \times Nb} \end{bmatrix}$ , which is  $N \times N(b + 1)$ .

The dynamics of  $\mathbf{y}_t$  are uniquely determined by  $\boldsymbol{\phi}_t$  through (2). For example, if  $\boldsymbol{\phi}_t$  is  $(N \cdot k) \times 1$  and contains the first  $k$  lags of each element of  $\mathbf{y}_t$ , then  $\mathbf{y}_t$  is VAR( $k + 1$ ). Alternatively, if  $\boldsymbol{\phi}_t$  instead contains the first  $k$  lags of each element of  $\boldsymbol{\eta}_t$ , then  $\mathbf{y}_t$  is a VMA( $k + 1$ ). Those dynamics will in turn define (2) and the  $\mathbf{T}$  matrix; in the case of the VAR( $k + 1$ ) model mentioned above,  $\mathbf{T}$  is simply our matrix of autoregressive coefficients and  $\mathbf{R}$  determines the covariance matrix of our error terms  $\mathbf{R}\mathbf{R}'$ . Suitable definition of  $\boldsymbol{\phi}_t$  allows us to model a broad range of stationary and nonstationary vector processes; Harvey (1989) provides a wide array of examples including models of trend-cycle decompositions. In our Illustration below, we employ autoregressive processes (AR(2) and VAR(2)) for the dynamics of the true values.

When we consider data revisions, we distinguish between “true” values of  $\mathbf{y}_t$  and estimates (of various releases) published by statistical agencies. In what follows below, therefore, we will replace  $\mathbf{y}_t$  in the state vector  $\boldsymbol{\alpha}_t$  with  $\tilde{\mathbf{y}}_t$  and several other features of (1) and (2) will change. However, the role of  $\boldsymbol{\phi}_t$  will not; its only purpose is to capture the dynamics of the “true” values.

### 3.1 Adding noise

The simplest way to model data revisions is to assume that published series  $\mathbf{y}_t$  consist of “true” values  $\tilde{\mathbf{y}}_t$  plus error. In this case,  $l \geq 1$  and in (1)  $\mathbf{y}_t$  is now  $Nl \times 1$ , where the first  $l$  rows contain the releases of the first variable from  $y_{1t}^1$  through  $y_{1t}^l$ , the next  $l$  rows contain  $y_{2t}^1$  through  $y_{2t}^l$ , etc.;  $\tilde{\mathbf{y}}_t$  is  $N \times 1$ ;  $\boldsymbol{\phi}_t$  remains  $Nb \times 1$ ;  $\boldsymbol{\zeta}_t$  is  $N \times l$  and contains the measurement error associated with each corresponding element of  $\mathbf{y}_t$ ;  $\boldsymbol{\alpha}_t = [\tilde{\mathbf{y}}_t', \boldsymbol{\phi}_t', \boldsymbol{\zeta}_t']'$ , which is  $(N + Nb + Nl) \times 1 = N(l + b + 1) \times 1$ ; and  $\mathbf{Z} = \begin{bmatrix} \mathbf{I}_N \otimes \boldsymbol{\nu}_l & \mathbf{0}_{Nl \times Nb} & \mathbf{I}_N \otimes \boldsymbol{\nu}_l \end{bmatrix}$ , which is  $Nl \times N(l + b + 1)$ .

We may now write (2) more explicitly with

- $\mathbf{T} = \begin{bmatrix} \mathbf{T}_\phi & \mathbf{0}_{N(b+1) \times Nl} \\ \mathbf{0}_{Nl \times N(b+1)} & \mathbf{T}_{Noise} \end{bmatrix}$ , where  $\mathbf{T}_\phi$  is  $N(b + 1) \times N(b + 1)$  and so is conformable with  $[\tilde{\mathbf{y}}_t', \boldsymbol{\phi}_t']'$ . Its elements are precisely those which we would

have in the above case where only true values are observed.  $\mathbf{T}_{Noise}$  is  $Nl \times Nl$  and so is conformable with  $\boldsymbol{\zeta}_t$ .  $\mathbf{T}_{Noise} = \mathbf{0}_{Nl \times Nl}$  implies that measurement errors are independent across different calendar dates, while  $\mathbf{T}_{Noise} \neq \mathbf{0}_{Nl \times Nl}$  implies that measurement errors in adjacent periods will be correlated.<sup>2</sup>

- $\mathbf{R} = \begin{bmatrix} \mathbf{R}_\phi & \mathbf{0}_{N(b+1) \times Nl} \\ \mathbf{0}_{Nl \times N} & \mathbf{R}_{Noise} \end{bmatrix}$  where  $\mathbf{R}_\phi$  is  $N(b + 1) \times N(b + 1)$ . Its elements are precisely those which we would have in the above case where only true values

are observed.  $\mathbf{R}_{Noise}$  is  $Nl \times Nl$ . Its elements are precisely those which define the variance-covariance matrix of the measurement errors *across releases*. In the special case where measurement errors are uncorrelated across variables, this will be a block-diagonal matrix with  $N$  blocks of size  $l \times l$ .

- $\boldsymbol{\eta}_t = \begin{bmatrix} \tilde{\boldsymbol{\eta}}_t' & (\boldsymbol{\eta}_t^{Noise})' \end{bmatrix}'$  where  $\tilde{\boldsymbol{\eta}}$  is  $N(b + 1) \times 1$  and contains the time  $t$  innovations to the  $N$  true values of  $\tilde{\mathbf{y}}_t$ ;  $\boldsymbol{\eta}_t^{Noise}$  is  $Nl \times 1$  and contains the time  $t$  innovations for all the measurement errors in  $\mathbf{y}_t$ .

---

<sup>2</sup>In principle, we could allow for higher-order correlation in measurement errors across calendar time by stacking multiple lags of  $\boldsymbol{\zeta}_t$  into the state vector.

A common source of confusion in models of data revisions is distinction between correlations across *releases* (or vintages) and correlations across *time*. If the statistical agency tends to revise its initial estimates upwards over the course of several releases, then measurement errors are correlated *across releases*; successive releases tend to have negative measurement errors. On the other hand, if once a year the agency incorporates information from annual income tax returns and tends to revise all the quarterly or monthly estimates for the preceding year in the same direction, then measurement errors are correlated *across time*. Following JvN, we define  $\mathbf{y}_t$  to group all the various releases (for  $1, \dots, l$ ) for a given point in time  $t$ . This implies that the  $\mathbf{R}$  matrix captures correlations across *releases* while the  $\mathbf{T}$  matrix captures correlations across *time*.

If we think that successive revisions tend to improve the reliability of published series by tending to reduce measurement errors, we can incorporate this through further restrictions on  $\mathbf{R}_{Noise}$ . For example, in the simplest case where  $\mathbf{R}_{Noise}$  is diagonal, we could require that in each of our  $N$  blocks of  $l$  releases,  $i < j \iff \sigma_{k,i} > \sigma_{k,j} \forall k$  where  $\sigma_{k,n}$  is the diagonal entry in  $\mathbf{R}_{Noise}$  corresponding to the  $n$ th release of the  $k$ th variable.

The block-diagonal forms of  $\mathbf{R}$  and  $\mathbf{T}$  imply that measurement errors are independent of the true values  $\tilde{\mathbf{y}}_t$ . This is sufficient to ensure that measurement errors will be noise. However, when measurement errors are news, they must be correlated with  $\tilde{\mathbf{y}}$ , which in turn will require the introduction of some off-diagonal elements as we will see in the next section.

## 3.2 Adding news

### News versus noise

The modeling of measurement errors has two main traditions. The older and most widespread approach models measurement errors as noise; i.e. random errors which are orthogonal to true values ( $\tilde{\mathbf{y}}_t$ ). This implies for all releases  $i$  of a single variable

$y_t^i$  that

$$y_t^i = \tilde{y}_t + \zeta_t^i, \quad \text{cov}(\tilde{y}_t, \zeta_t^i) = 0. \quad (3)$$

This is precisely the case we saw above. The newer tradition, motivated by Mankiw, Runkle and Shapiro (1984), Mankiw and Shapiro (1986) and de Jong (1987), describes measurement errors as news. News errors imply that published data are optimal forecasts, so revisions are orthogonal to earlier releases and are not forecastable.<sup>3</sup> More precisely, we require that

$$\tilde{y}_t = y_t^i + \nu_t^i, \quad \text{cov}(y_t^i, \nu_t^i) = 0. \quad (4)$$

Of course, a direct implication of this last condition is that  $\text{cov}(\tilde{y}_t, \nu_t^i) \neq 0$ . Furthermore, if the information available to the statistical agency increases through time, the variance of the measurement errors  $\nu_t^i$  must be decreasing. Since  $\text{var}(\tilde{y}_t)$  is given, *decreases* in  $\text{var}(\nu_t^i)$  imply an offsetting *increase* in  $\text{var}(y_t^i)$ . This is precisely the opposite of the noise case, where *decreases* in  $\text{var}(\zeta_t^i)$  imply a corresponding *decrease* in  $\text{var}(y_t^i)$ .  $\text{cov}(y_t^i, \nu_t^i) = 0$  also implies that news  $\nu_t^i$  will be positively correlated with innovations in  $\tilde{y}_t$ . We accommodate this by adding off-diagonal elements to  $\mathbf{R}$ .

### A model of news

If measurement errors are news rather than noise, the only change required to the “noise” state-space model that we considered above is in  $\mathbf{R}$ . However, to avoid confusion we will relabel the following elements without changing their dimensions:  $\mathbf{T}_{Noise}$  will be renamed  $\mathbf{T}_{News}$ ;  $\zeta_t$  will be renamed  $\nu_t$  and partitioned as  $\nu_t = \begin{bmatrix} \tilde{\nu}_t' & (\nu_t^{News})' \end{bmatrix}$ ; and  $\mathbf{R}_\phi$  will now be partitioned so that  $\mathbf{R}_\phi = \begin{bmatrix} \mathbf{R}'_{\phi 1} & \mathbf{R}'_{\phi 2} \end{bmatrix}'$  with the first of its  $N$  rows in  $\mathbf{R}_{\phi 1}$  and the remaining  $Nb$  rows in  $\mathbf{R}_{\phi 2}$ .

---

<sup>3</sup>Sargent (1989) motivates this by modeling the statistical agency as solving a mean-squared error estimation problem by linearly projecting the variables they seek to estimate on the set of available information. As more information arrives, estimates become more precise. However, analogous to the case of rational expectations, data revisions will be unpredictable given the information used to construct the original estimates.

Also, whereas previously  $\mathbf{R} = \begin{bmatrix} \mathbf{R}_\phi & \mathbf{0}_{N(b+1) \times Nl} \\ \mathbf{0}_{Nl \times N(b+1)} & \mathbf{R}_{Noise} \end{bmatrix}$ , we now redefine  $\mathbf{R}$  as

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{\phi 1} & \mathbf{R}_{News} \\ \mathbf{R}_{\phi 2} & \mathbf{0}_{Nb \times Nl} \\ \mathbf{0}_{Nl \times N(b+1)} & -\mathbf{U} \cdot \text{diag}(\vec{\sigma}_\nu) \end{bmatrix} \quad (5)$$

where

- $\vec{\sigma}_\nu \equiv [\sigma_{\nu 1}, \sigma_{\nu 2}, \dots, \sigma_{\nu Nl}]'$  is a  $Nl \times 1$  vector containing the standard deviations of news shocks for every release and every variable; the first  $l$  entries contain the news shocks for releases 1 through  $l$  of variable 1, then next  $l$  entries contain the news shocks for variable 2, etc.;
- $\mathbf{R}_{News} = (\mathbf{I}_N \otimes \mathbf{U}_l) \cdot \vec{\sigma}_\nu$ , an  $N \times Nl$  matrix; which is block diagonal, with  $N$  blocks each consisting of a row vector of length  $l$ ; the  $j$ th block contains the standard deviation of the  $l$  news shocks for variable  $j$ ;
- $\mathbf{U}_l$  is a  $l \times l$  matrix with zeros below the main diagonal and ones everywhere else;
- $\mathbf{U} \equiv \mathbf{I}_N \otimes \mathbf{U}_l$  is a  $Nl \times Nl$  matrix;
- $\text{diag}(\vec{\sigma}_\nu)$  is a  $Nl \times Nl$  matrix with  $\vec{\sigma}_\nu$  on its main diagonal and zeros everywhere else.

The effect of  $\mathbf{R}_{News}$  is that it accumulates, for each of the  $N$  variables, all the  $l$  different news shocks and adds them to the innovations affecting that variable's true values  $\tilde{y}_t^i$ . This off-diagonal matrix thereby ensures that revisions will be correlated with innovations to true values. In contrast,  $-\mathbf{U} \cdot \text{diag}(\vec{\sigma}_\nu)$  then removes some or all of these innovations from the various releases, with earlier releases having more information removed. This means that there is no need to impose restrictions on

the elements of  $[\sigma_{\nu 1}, \sigma_{\nu 2}, \dots, \sigma_{\nu Nl}]$  in order to ensure that later releases are more precise or more volatile than earlier releases. Since later releases contain more of the shocks to true values than earlier releases, this also ensures that revisions will tend to be positively correlated with true values, as we discussed above.<sup>4</sup>

In news models, note that for each variable the number of free parameters in  $\mathbf{R}$  grows linearly with the number of releases  $l$ , whereas with noise models it may grow proportional to  $l^2$ . This simply reflects the fact that the assumption of news imposes more restrictions on the behaviour of data revisions than does the assumption of noise.

### 3.3 A model of news and noise

A common empirical finding is that data revisions appear to be neither pure news nor pure noise. We therefore allow for revisions to be the sum of both news and noise components. This is done by expanding the state vector to contain both types of measurement errors and conformably expanding the rest of our system matrices. Our the state vector  $\boldsymbol{\alpha}_t$  then becomes

$$\boldsymbol{\alpha}_t = [\tilde{\mathbf{y}}'_t, \boldsymbol{\phi}'_t, \boldsymbol{\nu}'_t, \boldsymbol{\zeta}'_t]', \quad (6)$$

where the definitions are as above and the four components are of length  $N$ ,  $Nb$ ,  $Nl$  and  $Nl$  respectively. We conformably partition

$$\mathbf{Z} = \left[ \tilde{\mathbf{Z}}, \mathbf{0}_{Nl \times Nb}, \mathbf{I}_{Nl}, \mathbf{I}_{Nl} \right] \quad (7)$$

---

<sup>4</sup>Dungey et al. (2014) discuss how the shock correlations imposed in the state-space representation of news shocks relate to the state-space representation of the Beveridge-Nelson decomposition, while Dungey et al. (2013) note that such shock correlations imply that the “smoothed” estimates of the true values  $\tilde{\mathbf{y}}_t$  produced by usual Kalman filter recursions will be more variable than the corresponding “filtered” estimates.

where  $\tilde{\mathbf{Z}} = \mathbf{I}_N \otimes \boldsymbol{\nu}_l$ ; as before, this is a  $Nl \times Nl$  block-diagonal matrix with a vector of  $l$  ones on the main diagonal. The measurement equation (1) then simplifies to

$$\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t = \tilde{\mathbf{y}}_t + \boldsymbol{\nu}_t + \boldsymbol{\zeta}_t = \text{'Truth'} + \text{'News'} + \text{'Noise'}.$$

Moving on to the Transition equation, we analogously partition matrix  $\mathbf{T}$  as

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_\phi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{News} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T}_{Noise} \end{bmatrix}, \quad (8)$$

where  $\mathbf{T}_\phi$ ,  $\mathbf{T}_{News}$  and  $\mathbf{T}_{Noise}$  are as previously defined.

$\mathbf{R}$  becomes a  $N(1+b+2l) \times N(1+b+2l)$  matrix partitioned as follows

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{\phi 1} & \mathbf{R}_{News} & \mathbf{0} \\ \mathbf{R}_{\phi 2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{U} \cdot \text{diag}(\vec{\sigma}_\nu) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{Noise} \end{bmatrix}, \quad (9)$$

where  $\mathbf{R}_{\phi 1}$ ,  $\mathbf{R}_{\phi 2}$ ,  $\mathbf{R}_{News}$ ,  $\mathbf{R}_{Noise}$ ,  $\mathbf{U}$  and  $\text{diag}(\vec{\sigma}_\nu)$  are as defined above. Finally, we partition the error vector associated with the transition equation as  $\boldsymbol{\eta}_t = [\boldsymbol{\eta}'_{et}, \boldsymbol{\eta}'_{\nu t}, \boldsymbol{\eta}'_{\zeta t}]'$ , where  $\boldsymbol{\eta}_{et}$  (with length  $1+b$ ) refers to errors associated with the true values, and  $\boldsymbol{\eta}_{\nu t}$  and  $\boldsymbol{\eta}_{\zeta t}$  (both of length  $l$ ) are the errors for news and noise, respectively.

In our Illustration in Section 5 below we allow correlated news and noise innovations for the  $l$  releases of the  $N$  variables in the system. Section 3.5 discusses the modeling of correlated shocks in more detail.

The transition equation assumes that news and noise terms only enter the dynamic equations for the variables to which they belong, i.e. news shocks to the first variable enter the the news equation of the first variable in the system. This is the assumption we make in the Illustration below. More complex news-noise structures

are of course also possible.

### 3.4 Factor structures and data reconciliation

All of the above models of data revision have a factor structure which relates a single underlying true value  $\tilde{y}_{i,t}$  to a vector of  $l$  different releases. Conventional factor models identify the underlying factors by assuming that deviations from the factor are orthogonal to the factor itself. This maximizes the explanatory power of the underlying factor and also exactly corresponds to the properties of our “noise” errors. “News” errors represent an alternative formulation of the factor structure related to Beveridge-Nelson decompositions.<sup>5</sup> In both cases we have thus far assumed that we have as many underlying true values (i.e. factors) as we have distinct series. However, the framework may also be easily adapted to cases whether this is no longer true.

Economists are sometimes faced with the problem of reconciling conflicting official estimates. Some countries (including the USA) publish distinct estimates of Gross Domestic Product and Gross Domestic Income. Conceptually these should be identical, but recent work has highlighted some important differences.<sup>6</sup> Other examples include reconciling estimates produced from different methodologies, such as those produced by surveys and censuses, or plant-level and firm-level surveys, or household and establishment surveys. These are situations in which we have fewer underlying true values than we have reported series.

More generally, we can think of having  $N$  elements in  $\tilde{\mathbf{y}}_t$ , but that economic theory implies a set of linear restrictions on those values such that

$$\mathbf{W} \cdot \tilde{\mathbf{y}}_t = \mathbf{w}$$

Such restrictions can easily be incorporated in standard state-space systems by aug-

---

<sup>5</sup>See Dungey et al. (2014).

<sup>6</sup>See e.g. Fixler and Nalewaik (2009) and Aruoba et al. (2012, 2013).

menting the measurement equation (1) to give

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{w} \end{bmatrix} = \begin{bmatrix} \mathbf{Z} \\ \mathbf{W} \end{bmatrix} \tilde{\mathbf{y}}_t. \quad (10)$$

This formulation preserves  $\tilde{\mathbf{y}}$  as a full  $N$ -dimension vector which is now subject to a number of linear constraints. In simple cases, an alternative formulation would be to reduce the dimension of  $\tilde{\mathbf{y}}$  and directly incorporate the factor restrictions in the specification of  $\mathbf{Z}$ . For example, consider the case where the last  $p$  of our  $N$  variables are simply alternative measures of the same underlying economic concept. In that case, when we construct our  $\mathbf{Z} = [\tilde{\mathbf{Z}}, \mathbf{0}_{Nl \times Nb}, \mathbf{I}_{Nl}, \mathbf{I}_{Nl}]$ , we can replace  $\tilde{\mathbf{Z}} = \mathbf{I}_N \otimes \boldsymbol{\iota}_l$  with  $\tilde{\mathbf{Z}}^* \equiv \begin{bmatrix} \mathbf{I}_{N-p} \otimes \boldsymbol{\iota}_l & \mathbf{0}_{l(N-p) \times 1} \\ \mathbf{0}_{pl \times (N-p)} & \boldsymbol{\iota}_{pl} \end{bmatrix}$ . This causes all  $l$  releases of the last  $p$  variables to share the same true values.<sup>7</sup> Obviously, this also reduces the dimension of  $\tilde{\mathbf{y}}_t$  from  $N$  to  $N - p + 1$ , and may (or may not) reduce the dimension of  $\phi_t$ . However, this need not reduce the dimensions of  $\boldsymbol{\nu}_t$  or  $\boldsymbol{\zeta}$ , as the measurement errors affecting each variable will typically continue to be unique to that series.

Note that in such a system, the news shocks of several variables are all shared by a single element of  $\tilde{\mathbf{y}}_t$ . To the extent that innovations in different published series are correlated, this means that news shocks must be correlated across variables, a possibility that we have not previously considered. We discuss the modeling of such systems of correlated shocks in the next section.

### 3.5 Correlated shocks

Correlated shocks to true values, news or noise change the variance-covariance of the innovations  $\mathbf{R}\boldsymbol{\eta}$  in the state equation. We can model this in two ways: (i) taking aboard the correlation in matrix  $\mathbf{R}$  to get  $\mathbf{R}^+$ , without changing the properties of

---

<sup>7</sup>A simple extension would be the case were we assume that all  $p$  variables are the same up to a  $p \times 1$  vector of scaling factors  $\vec{\lambda}$ . In that case, we need only replace  $\boldsymbol{\iota}_{pl}$  with  $\vec{\lambda} \otimes \boldsymbol{\iota}_l$ .

the shocks, i.e.  $\boldsymbol{\eta}_t \sim N(\mathbf{0}, \mathbf{I}_{N_r})$ ; or (ii) keeping the same matrix  $\mathbf{R}$  and allowing for correlation between the elements of  $\boldsymbol{\eta}$  through a general variance-covariance matrix  $\mathbf{Q}$ , i.e.  $\boldsymbol{\eta}_t^+ \sim N(\mathbf{0}, \mathbf{Q})$ . In both cases the variance-covariance matrix of the innovations becomes  $\boldsymbol{\Omega}^+$ , i.e.  $E(\mathbf{R}^+ \boldsymbol{\eta} \boldsymbol{\eta}' (\mathbf{R}^+)' ) = E(\mathbf{R} \boldsymbol{\eta}^+ (\boldsymbol{\eta}^+)' \mathbf{R}') = \boldsymbol{\Omega}^+$ , which makes the specifications observationally equivalent. In our Illustration below, we apply the second method to deal with correlated shocks in our implementation of the method.

We now describe how to incorporate news shocks that are correlated across variables in the matrix  $\mathbf{R}^+$ . More formally, let  $H_\tau^j \equiv \{y_{jt}^\tau, y_{jt}^{\tau-1}, y_{jt}^{\tau-2}, \dots\}$  be the set of all current and past releases of some scalar time series  $y_{jt}$  and let  $H_\tau^{\mathbf{y}} \equiv \{\mathbf{y}_t^\tau, \mathbf{y}_t^{\tau-1}, \mathbf{y}_t^{\tau-2}, \dots\}$  be the set of all current and past releases of some vector time series  $\mathbf{y}$  whose  $j$ th element is  $y_{jt}$ . In this context, we may define “news” in two different ways. We refer to measurement errors as *Univariate News* if and only if, for some series  $j$ , current and past estimates of the series do not help us predict its future revisions. Mathematically, we may write this as  $E(y_{jt}^{\tau+i} - y_{jt}^\tau | H_\tau^j) = 0 \forall t, \tau, j$  and  $\forall i > 0$ . This is the definition used above in Section 3.2. A more restrictive case will be called *Multivariate News*: in this case the revisions of series  $j$  cannot be predicted by current or past estimates of *any* of the series in  $\mathbf{y}_t$ . We may define this more formally as the case where  $E(y_{jt}^{\tau+i} - y_{jt}^\tau | H_\tau^{\mathbf{y}}) = 0 \forall t, \tau, j$  and  $\forall i > 0$ .

Note that

- the only difference between Univariate and Multivariate News is the information set on which we condition the expectations.
- $H_\tau^j \subset H_\tau^{\mathbf{y}}$ . Therefore, if the Multivariate News condition is satisfied, the Univariate News condition will also be satisfied.
- with Multivariate News, news shocks may only be correlated across variables if they are both shocks to the same release  $\tau$ .

We now consider the problem of incorporating Multivariate News into our model. In Section 3.3 above, we specified the final term in the state equation to have the form

$$\begin{bmatrix} \mathbf{R}_{\phi 1} & \mathbf{R}_{News} & \mathbf{0} \\ \mathbf{R}_{\phi 2} & \mathbf{0} & \mathbf{0} \\ 0 & -\mathbf{U} \cdot \text{diag}(\vec{\sigma}_\nu) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}_{Noise} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\eta}_{et} \\ \boldsymbol{\eta}_{\nu t} \\ \boldsymbol{\eta}_{\zeta t} \end{bmatrix}$$

Correlated news shocks requires only that we modify the columns of  $\mathbf{R}$  corresponding to  $\boldsymbol{\eta}_{\nu t}$ . This can be done as follows:

- $\boldsymbol{\lambda}_{ij}$  is a row vector of length  $l$  which captures the comovements of innovations to the estimates of variable  $j$  and variable  $i$ . This is a  $1 \times l$  vector since the comovement may vary across the  $l$  different releases of the series.
- $\text{diag}(\boldsymbol{\lambda}_{ij}) \equiv$  a  $l \times l$  matrix with the elements of  $\boldsymbol{\lambda}_{ij}$  on its diagonal.

- $\mathbf{R}_{News} = \begin{bmatrix} \boldsymbol{\lambda}_{11} & \dots & \boldsymbol{\lambda}_{1N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\lambda}_{N1} & \dots & \boldsymbol{\lambda}_{NN} \end{bmatrix}$ , a  $N \times Nl$  matrix.

- $\mathbf{U}_{News} = \begin{bmatrix} -\mathbf{U}_l \cdot \text{diag}(\boldsymbol{\lambda}_{11}) & \dots & \dots & -\mathbf{U}_l \cdot \text{diag}(\boldsymbol{\lambda}_{1N}) \\ \mathbf{0}_{l \times l} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0}_{l \times l} & \dots & \mathbf{0}_{l \times l} & -\mathbf{U}_l \cdot \text{diag}(\boldsymbol{\lambda}_{NN}) \end{bmatrix}$  where  $\mathbf{U}_l$  is an  $l \times l$

upper-triangular matrix of ones (as above).

In future research we will also implement this  $\mathbf{R}^+$  matrix to model correlated news and investigate implications for testing for news in a multivariate setting.

## 4 Bayesian estimation

To estimate the model we use standard Gibbs sampling methods (Geman and Geman 1984; Gelfand and Smith 1990; Kim and Nelson 1999). The Gibbs sampler applied

in this paper proceeds as follows. Let  $\Psi$  contain all parameters of the model and  $U$  all latent variables. Given arbitrary initial values  $\Psi^0$  and  $U^0$ , draws for  $\Psi$  and  $U$  are obtained from the following conditional distributions  $\{\Psi^1 \sim p(\Psi|U^0), U^1 \sim p(U|\Psi^1)\}$ ,  $\{\Psi^2 \sim p(\Psi|U^1), U^2 \sim p(U|\Psi^2)\}$ ,  $\dots$ ,  $\{\Psi^w \sim p(\Psi|U^{w-1}), U^w \sim p(U|\Psi^w)\}$ . It can be shown that under mild conditions the resulting Gibbs sequence  $\{\Psi^w, U^w\}$  converges (in distribution) to the true joint density at a geometric rate in  $w$  (Geman and Geman 1984). To obtain draws for the latent state variables in  $U$  the approach of Carter and Kohn (1994) is applied (see Kim and Nelson 1999). The parameters in  $\Psi$  are drawn from a multivariate normal distribution and an inverted Wishart distribution.

We cycle through 100,000 Gibbs iterations, discarding the first 80,000 draws as burn-in and saving every 10th draw. Convergence is checked using recursive mean plots of the parameters.

## Priors

We begin with the multivariate normally distributed prior for the system matrix  $\mathbf{T}$ :

$$\mathbf{T}_{prior} \sim N(\bar{\Psi}, \bar{\mathbf{V}}_T).$$

For the mean of the parameters we assume that  $\bar{\mathbf{T}}$  is an  $N^{*2}$  vector of zeros, with  $N^* \equiv N + Nb + 2Nl$  and that the variance covariance matrix  $\bar{\mathbf{V}}_T$  is a  $N^{*2} \times N^{*2}$  identity matrix times  $\mathbf{A}$ , where  $\mathbf{A}$  is a  $N^{*2} \times N^{*2}$  diagonal matrix with  $a_i, i = 1, \dots, N^{*2}$  on the diagonal. For the coefficients where zero restrictions are imposed we choose  $a$  to be close to zero. For the VAR coefficients of the true values of imports and exports we choose  $a = 10$  for all the other AR coefficients we choose  $a = 1$ .

The second group of parameters of our state-space form are the parameters in the  $\mathbf{R}$  matrix of the state equation, which enter the model in the form of the variance-covariance matrix  $E(\mathbf{R}\eta^+(\eta^+)' \mathbf{R}') = \mathbf{\Omega}^+$ . Rewriting  $\mathbf{R}\eta^+$  by putting all

parameters in the shocks allows us to draw from the distributions of the shocks. The prior on the variance covariance matrix of the shocks in the state equation follows an inverted Wishart distribution and can be expressed as:

$$\mathbf{Q}_{prior} \sim IW(\bar{\mathbf{Q}}, \delta),$$

where  $\delta \equiv \dim(\mathbf{T}) + 2$  and

$$\bar{\mathbf{Q}} = \begin{bmatrix} \bar{\mathbf{Q}}_{xy} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{Q}}_{\nu\zeta} \end{bmatrix}. \quad (11)$$

In our Illustration below  $\bar{\mathbf{Q}}_{xy}$  is a  $2 \times 2$  matrix for the prior of the variance-covariance of errors of the true values, which is an identity matrix in case of an AR model and has ones on the diagonal and 0.9 as off-diagonal elements for the VAR of true values, and  $\bar{\mathbf{Q}}_{\nu\zeta}$  is a  $2Nl \times 2Nl$  matrix for the prior of the variance-covariance matrix of the measurement errors, which is an identity matrix both in case of uncorrelated and correlated news and noise innovations.

## 5 Illustration

We illustrate our multivariate data revision modeling framework with Swiss current account data. After a description of the data, we present properties of data revisions of Swiss imports and exports and the current account, and evidence of possible multivariate relationships. We estimate several alternative specifications of the model, assuming AR(2) processes for the true values of imports and exports and a VAR(2) system for the dynamics of their true values, correlated news and noise across time, and uncorrelated and correlated news and noise shocks across variables.

## 5.1 Data and data properties

In Switzerland current account figures are collected by the Swiss National Bank (SNB) and published in its Monthly Bulletins ('Statistische Monatshefte'). Current account information is provided for income (exports), expenditures (imports) and net exports (exports minus imports).

Our real-time data set consists of monthly vintages with quarterly data of these three series. The first vintage, published in August 1995, covers the 1984Q1–1995Q2 period, while our last vintage, published in September 2012, has data for the 1984Q1–2012Q1 period. The data are kindly provided by the Swiss National Bank (SNB).

We observe comprehensive revisions in the vintages published in August 2004, July 2005, July 2007, July 2008, December 2008. The first two can be explained by the introduction of SNA93 and ESA95. In all five cases, the SNB revised the data back to 1984Q1. In order to mitigate the effects of these comprehensive revisions, most authors would use growth rates (Croushore 2006). However, as shown by Siklos (2008) and Knetsch and Reimers (2009), this is not optimal as comprehensive revisions behave differently from other revisions. Therefore, we use levels in this paper and deal with these comprehensive revisions more directly. This also allows us to deal with adding-up restrictions in a straightforward manner.

Unlike regular revisions, comprehensive revisions affect the vintage from the beginning to the end. To eliminate the five comprehensive revisions, we extrapolate the revisions of the older part of the vintage (from 1984) to the most recent observations. For this an AR(4) process is assumed. The part not explained by this extrapolated AR(4) process, i.e. the difference between the observed observations in the vintage and the extrapolated values, is treated as non-comprehensive revisions.

## Descriptive statistics

We distinguish between revisions after 3, 12 and 36 months. Table 1 shows that the first revisions on both the export and import side of the Swiss current account are relatively small, but tend to accumulate over time. Both expenditures (imports) and income (exports) are revised upwards on average. With a cumulative average revision of, respectively 4.2 and 2.6 percent, after three years, these are not trivial for the evaluation of economic conditions in a small open economy as Switzerland. Except for the first revision the means reported in Table 1 all differ significantly from zero at the one percent level (not shown).

Table 1: Descriptive statistics of the revision process

	Mean	% $\Delta$ to 1st rel.	Std.Dev.	Min	Max
Expenditures (imports)					
First release	62,364.61		19,689.18	36,512	106,551
Revisions after 1 quarter	452.75	0.7%	1,056.56	-1,099	3,250
Revisions after 1 year	1,740.98	2.8%	2,770.74	-3,480	11,510
Revisions after 3 years	2,609.72	4.2%	3,418.27	-3,372	12,385
Income (exports)					
First release	74,282.11		23,105.00	42,881	121,229
Revisions after 1 quarter	624.00	0.8%	1,416.46	-1,481	4,761
Revisions after 1 year	1,325.42	1.8%	3,488.96	-10,052	10,344
Revisions after 3 years	1,959.68	2.6%	4,485.94	-11,198	10,511

*Notes:* Results are based upon 57 observations. Levels and revisions are shown in millions of CHF.

The left panel of Figure 1 in the Introduction shows revisions of Swiss imports and exports after one year, as percentages of the first releases. The correlation between these revisions is quite high, which suggests the relevance of using our multivariate data revision modeling approach for this data set.

## 5.2 Estimation of our state-space forms

We use the 1st, the 12th and 36th releases in our estimations which allows us to capture the effects of the initial and annual (seasonal) revisions. The series run from 1995Q2–2009Q2, so we have 57 quarterly observations. Observed series are normalised globally, i.e. a scalar mean is subtracted and all series are scaled by the same standard deviation. Consequently, a constant term is dropped from the state-space form.

We set up the system assuming correlation across time in news and noise, i.e. diagonal matrices  $\mathbf{T}_3$  and  $\mathbf{T}_4$  enter the transition equation. Under the assumption that the true values of imports  $x$  and exports  $y$  follow a VAR(2) process

$$\begin{bmatrix} \tilde{x}_t \\ \tilde{y}_t \end{bmatrix} = \Theta_1 \begin{bmatrix} \tilde{x}_{t-1} \\ \tilde{y}_{t-1} \end{bmatrix} + \Theta_2 \begin{bmatrix} \tilde{x}_{t-2} \\ \tilde{y}_{t-2} \end{bmatrix} + \begin{bmatrix} \eta_{xet} \\ \eta_{yet} \end{bmatrix},$$

the multivariate data revision model with news, noise, and correlated measurement errors across time can be expressed as the measurement equation

$$\begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ y_t^1 \\ y_t^2 \\ y_t^3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\iota}_3 & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 1} & \boldsymbol{\iota}_3 & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} \tilde{x}_t \\ \tilde{y}_t \\ \tilde{x}_{t-1} \\ \tilde{y}_{t-1} \\ \boldsymbol{\nu}_t \\ \boldsymbol{\zeta}_t \end{bmatrix},$$

and the transition equation

$$\begin{bmatrix} \tilde{x}_t \\ \tilde{y}_t \\ \tilde{x}_{t-1} \\ \tilde{y}_{t-1} \\ \boldsymbol{\nu}_t \\ \boldsymbol{\zeta}_t \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Theta}_1 & \boldsymbol{\Theta}_2 & \mathbf{0}_{2 \times 6} & \mathbf{0}_{2 \times 6} \\ \mathbf{I}_2 & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 6} & \mathbf{0}_{2 \times 6} \\ \mathbf{0}_{6 \times 2} & \mathbf{0}_{6 \times 2} & \mathbf{T}_3 & \mathbf{0}_{6 \times 6} \\ \mathbf{0}_{6 \times 2} & \mathbf{0}_{6 \times 2} & \mathbf{0}_{6 \times 6} & \mathbf{T}_4 \end{bmatrix} \begin{bmatrix} \tilde{x}_{t-1} \\ \tilde{y}_{t-1} \\ \tilde{x}_{t-2} \\ \tilde{y}_{t-2} \\ \boldsymbol{\nu}_{t-1} \\ \boldsymbol{\zeta}_{t-1} \end{bmatrix} +$$

$$\begin{bmatrix} 1 & 0 & \sigma_{x\nu_1} & \sigma_{x\nu_2} & \sigma_{x\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \sigma_{y\nu_1} & \sigma_{y\nu_2} & \sigma_{y\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sigma_{x\nu_1} & -\sigma_{x\nu_2} & -\sigma_{x\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sigma_{x\nu_2} & -\sigma_{x\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sigma_{x\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sigma_{y\nu_1} & -\sigma_{y\nu_2} & -\sigma_{y\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sigma_{y\nu_2} & -\sigma_{y\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sigma_{y\nu_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{x\zeta_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{x\zeta_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{x\zeta_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{y\zeta_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{y\zeta_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{y\zeta_3} \end{bmatrix} \begin{bmatrix} \eta_{xet} \\ \eta_{yet} \\ \eta_{x\nu_1t} \\ \eta_{x\nu_2t} \\ \eta_{x\nu_3t} \\ \eta_{y\nu_1t} \\ \eta_{y\nu_2t} \\ \eta_{y\nu_3t} \\ \eta_{x\zeta_1t} \\ \eta_{x\zeta_2t} \\ \eta_{x\zeta_3t} \\ \eta_{y\zeta_1t} \\ \eta_{y\zeta_2t} \\ \eta_{y\zeta_3t} \end{bmatrix},$$

where  $\begin{pmatrix} \eta_{xet} \\ \eta_{yet} \end{pmatrix} \sim \begin{pmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \end{pmatrix}$ .

Rewriting  $\mathbf{R}\boldsymbol{\eta}$  is convenient for working out the distributions of the errors:

$$\mathbf{R}\boldsymbol{\eta} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \eta_{xet} \\ \eta_{yet} \\ \sigma_{x\nu_1}\eta_{x\nu_1t} \\ \sigma_{x\nu_2}\eta_{x\nu_2t} \\ \sigma_{x\nu_3}\eta_{x\nu_3t} \\ \sigma_{y\nu_1}\eta_{y\nu_1t} \\ \sigma_{y\nu_2}\eta_{y\nu_2t} \\ \sigma_{y\nu_3}\eta_{y\nu_3t} \\ \sigma_{x\zeta_1}\eta_{x\zeta_1t} \\ \sigma_{x\zeta_2}\eta_{x\zeta_2t} \\ \sigma_{x\zeta_3}\eta_{x\zeta_3t} \\ \sigma_{y\zeta_1}\eta_{y\zeta_1t} \\ \sigma_{y\zeta_2}\eta_{y\zeta_2t} \\ \sigma_{y\zeta_3}\eta_{y\zeta_3t} \end{bmatrix},$$

where the errors associated with the true values  $\begin{bmatrix} \eta_{xet} \\ \eta_{yet} \end{bmatrix} \sim \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \right)$  and the other error terms follow a multivariate normal distribution with mean zero and variance  $\sigma_i^2$ , where  $i = x\nu_1, x\nu_2, x\nu_3, y\nu_1, y\nu_2, y\nu_3, x\zeta_1, x\zeta_2, x\zeta_3, y\zeta_1, y\zeta_2, \text{ and } y\zeta_3$ .

If one assumes univariate AR(2) processes for the dynamics of the true values of imports and exports, the specification stays the same apart from the matrices  $\boldsymbol{\Theta}_1$  and  $\boldsymbol{\Theta}_2$  and the variance-covariance matrix of the errors to the true values of imports and exports which in this case will be diagonal. In other words,  $\theta_{1,12} = \theta_{2,12} = \theta_{1,21} = \theta_{2,21} = \sigma_{xy} = \sigma_{yx} = 0$ .

Correlated news innovations across variables can be modeled as the shocks between

the true values of exports and imports in the VAR, i.e.

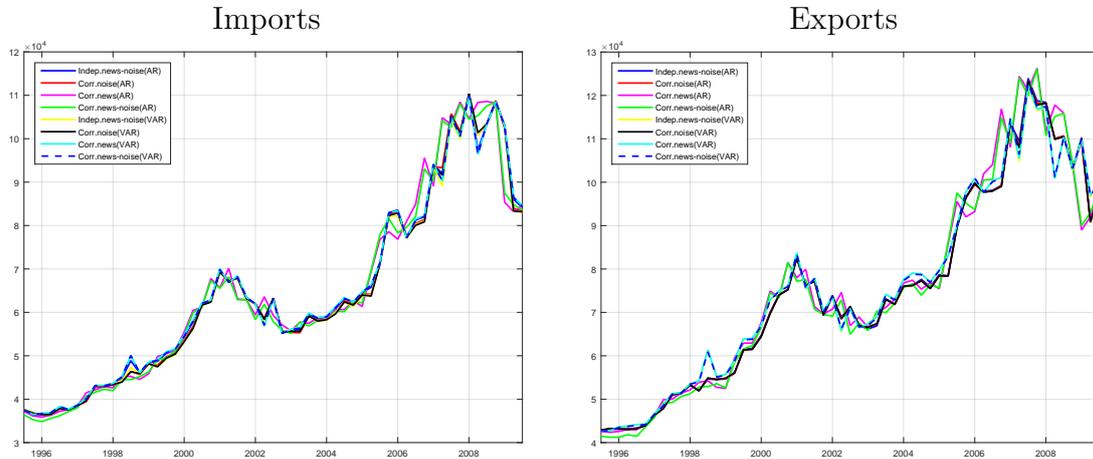
$$\begin{bmatrix} \eta_{x\nu_i t} \\ \eta_{y\nu_i t} \end{bmatrix} \sim \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{x\nu_i x\nu_i} & \sigma_{x\nu_i y\nu_i} \\ \sigma_{x\nu_i y\nu_i} & \sigma_{y\nu_i y\nu_i} \end{bmatrix} \right), \quad i = 1, 2, 3.$$

Similar expressions hold for correlated noise shocks, and correlated news and noise shocks.

## Results

We estimate both AR and VAR models for cases with independent news and noise innovations, correlated news, correlated noise and correlated news and noise.<sup>8</sup> Estimation results of all 8 model specifications are listed in Tables A.1–A.2 below. Note that in these two tables parameters  $T(3, 3), \dots, T(8, 8)$  refer to the diagonal elements of  $\mathbf{T}_3$ , and  $T(9, 9), \dots, T(14, 14)$  to the diagonal elements of  $\mathbf{T}_4$ .

Figure 2: Estimated true values of Swiss imports



These outcomes are not very informative. Instead we show the estimated true values of imports and exports in Figure 2 for all specifications distinguished. Patterns of true values in imports and exports look similar, but non-identical. The

<sup>8</sup>All model specifications contain spillovers. The Appendix of JvN (2007) describes sufficient conditions for the identification of JvN, which amounts to having sufficient dynamics for the true values and the measurement errors. It is generally speaking easier to show identification in specifications with spillovers.

figure does not allow us to answer the question which specification, AR or VAR, uncorrelated or correlated news or noise, produces the best fit.

Table 2 compares the different specifications on the basis of the Bayesian Information Criterion (BIC).<sup>9</sup> The table shows that the univariate models have lower BIC values than multivariate models. In addition, models with independent news and noise shocks are less supported by the data than models with correlated news and/or noise. The best model according to the BIC is the VAR(2) model with correlated news and noise. Apparently, not only correlation between true values of imports and exports matters in the multivariate data revision model, but also the spillover of news and noise.

Table 2: Model comparison

	BIC
Indep. news and noise (AR)	-115.63
Corr. news (AR)	-211.10
Corr. noise (AR)	-158.76
Corr. news and noise (AR)	-219.78
Indep. news and noise (VAR)	-200.96
Corr. news (VAR)	-238.21
Corr. noise (VAR)	-242.95
Corr. news and noise (VAR)	-252.46

Table 3 shows the correlations between news innovations in imports and exports for the three releases that we distinguish in the eight different model specifications. Correlations between news innovations in imports and exports are very high and significant in the univariate models with correlated news and correlated news and noise for all three releases. In the multivariate models in which innovations can also propagate through the correlated true' values the correlations between innovations in imports and exports are much lower and even absent for the 36th release.

<sup>9</sup>We also compared the different models on the basis of marginal likelihoods, which are computed using the procedure described in Chib (1995). As is well-known, marginal likelihoods depend on the choice of the prior and are therefore less or not conclusive.

Table 3: Correlation between news innovations in imports and exports

	First release		12th release		36th release	
	Mean	Std.	Mean	Std.	Mean	Std.
Indep. news and noise (AR)	0.21	0.11	0.09	0.14	0.01	0.13
Corr. noise (AR)	0.02	0.12	0.01	0.14	0.01	0.14
Corr. news (AR)	0.91	0.04	0.97	0.01	0.99	0.00
Corr. news and noise (AR)	0.86	0.05	0.97	0.01	0.99	0.01
Indep. news and noise (VAR)	0.21	0.07	0.12	0.15	0.00	0.13
Corr. noise (VAR)	-0.06	0.13	-0.01	0.13	0.00	0.13
Corr. news (VAR)	0.46	0.06	0.73	0.09	0.01	0.28
Corr. news and noise (VAR)	-0.12	0.30	0.58	0.38	0.01	0.28

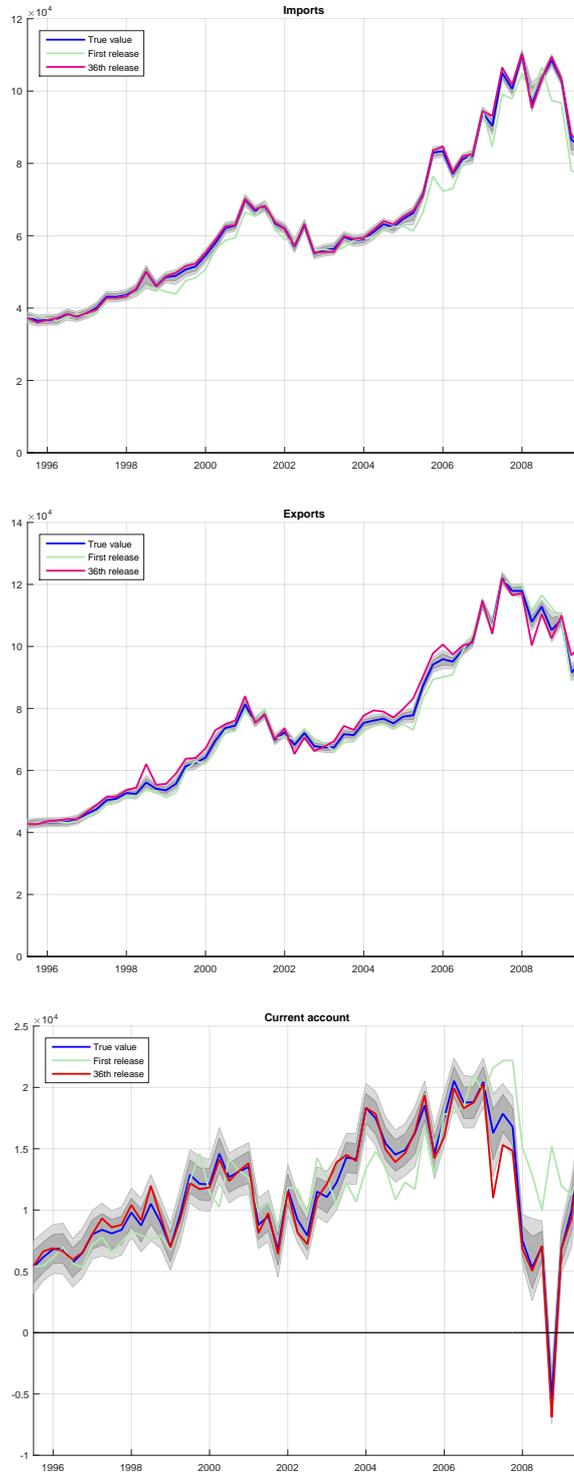
These conclusions do not carry over to noise innovations. Table 4 shows that correlations between noise innovations in imports and exports are high in the specifications with correlated noise for the first and the 36th release, but not for the 12th release. Correlations between noise correlations in the multivariate models show a similar pattern.

Table 4: Correlation between noise innovations in imports and exports

	First release		12th release		36th release	
	Mean	Std.	Mean	Std.	Mean	Std.
Indep. news and noise (AR)	0.09	0.13	0.06	0.13	0.32	0.17
Corr. noise (AR)	0.85	0.08	0.03	0.28	0.83	0.07
Corr. news (AR)	0.01	0.13	-0.01	0.13	-0.04	0.14
Corr. news and noise (AR)	0.80	0.28	-0.03	0.27	0.52	0.51
Indep. news and noise (VAR)	0.04	0.14	0.05	0.13	0.14	0.15
Corr. noise (VAR)	0.89	0.08	-0.05	0.28	0.78	0.08
Corr. news (VAR)	0.00	0.13	-0.01	0.13	-0.04	0.13
Corr. news and noise (VAR)	0.95	0.02	-0.02	0.26	-0.06	0.54

We summarize the outcomes of the best model, the VAR(2) model with correlated news and noise, in Figure 3, which shows the estimated true values, with 16th and 84th percentiles and 5th and 95th percentiles, together with first releases and 36th releases of imports, exports and the implied current account. The true values of imports and exports are estimated fairly precisely, i.e. with tight bounds. Bounds surrounding the true values of the current account are somewhat larger, as is to be expected. Releases after three years are generally within the bounds of the ‘true’ values. However, they do occasionally deviate from the ‘true’ values, see for example exports between 2004 and 2006. First releases deviate from the ‘true’ values especially around the years 2004–2005 and the Global Financial Crisis in 2008.

Figure 3: Model with spillovers and correlated news and noise



Red line: first release; blue line: true values. Dark gray area indicates 16th and 84th percentiles, light gray area 5th and 95th percentiles.

## 6 Conclusion

This paper proposed a general state-space framework to model multivariate data revisions, i.e. revisions occurring in more than one variable at a time, allowing correlated true values and news and noise measurement errors. We motivated and illustrated our multivariate framework with Swiss current account data. The estimation results demonstrate that a multivariate approach does pay off for our data set on the Swiss current account. VAR systems for true values of imports and exports are better supported by the data than AR processes. Moreover, we find strongest support for multivariate models with measurement errors that are correlated across time and across variables.

## Acknowledgements

This paper was written during visits of the first author to CIRANO and KOF Swiss Economic Institute, and of the second and the fourth author to the research school SOM of the University of Groningen. The hospitality and the support of these institutions, as well as that of CIREQ is gratefully acknowledged. We thank the Swiss National Bank (SNB) for kindly providing us with the data. Jacobs and van Norden would like to thank (without implicating) the late Prof. Arnold Zellner for encouraging them to pursue this topic. We thank Tim Hampton, Ataman Ozyildirim, Sebastiaan Pool and participants at several seminars and conferences for helpful comments.

## References

- Aruoba, S. Borağan, Francis X. Diebold, Jeremy Nalewaik, Frank Schorfheide, and Dongho Song (2012), “Improving GDP measurement: A forecast combination perspective”, in X. Chen and N. Swanson, editors, *Causality, Prediction, and Specification Analysis: Recent Advances and Future Directions: Essays in Honor of Halbert L. White Jr*, Springer, New York, 1–26.
- Aruoba, S. Borağan, Francis X. Diebold, Jeremy Nalewaik, Frank Schorfheide, and Dongho Song (2013), “Improving GDP measurement: A measurement-error perspective”, NBER Working Papers 18954, National Bureau of Economic Research, Inc.
- Carriero, Andrea, Mike Clements, and Ana B. Galvão (2014), “Bayesian multivariate vintage-based VARs”, *International Journal of Forecasting* [forthcoming].
- Carter, C. and R. Kohn (1994), “On Gibbs sampling for state space models”, *Biometrika*, **81**, 541–553.
- Chib, Siddhartha (1995), “Marginal likelihood from the Gibbs output”, *Journal of the American Statistical Association*, **90**, 1313–1321.
- Clements, Mike and Ana B. Galvão (2012), “Improving real-time estimates of output and inflation gaps with multiple-vintage models”, *Journal of Business & Economic Statistics*, **30**, 554–562.
- Croushore, Dean (2006), “Forecasting with real-time macroeconomic data”, in Graham Elliott, Clive W.J. Granger, and Allan Timmermann, editors, *Handbook of Economic Forecasting*, North-Holland, Amsterdam.
- Croushore, Dean (2011), “Frontiers of real-time analysis”, *Journal of Economic Literature*, **49**, 72–100.

- Cunningham, Alastair, Jana Eklund, Chris Jeffery, George Kapetanios, and Vincent Labhard (2012), “A state space approach to extracting the signal from uncertain data”, *Journal of Business & Economic Statistics*, **30**, 173–180.
- De Jong, Piet (1987), “Rational economic data revisions”, *Journal of Business & Economic Statistics*, **5**, 539–548.
- Dungey, Mardi, Jan P.A.M. Jacobs, Jing Tian, and Simon van Norden (2013), “On the correspondence between data revision and trend-cycle decomposition”, *Applied Economics Letters*, **20**, 316–319.
- Dungey, Mardi, Jan P.A.M. Jacobs, Jing Tian, and Simon van Norden (2015), “Trend in cycle or cycle in trend? New structural identifications for unobserved components models of U.S. real GDP”, *Macroeconomic Dynamics*, **19**, 776–790.
- Durbin, James and Siem Jan Koopman (2001), *Time Series Analysis by State Space Methods*, Oxford University Press, Oxford.
- Fixler, Dennis J. and Jeremy J. Nalewaik (2009), “News, noise, and estimates of the true unobserved state of the economy”, Mimeo, U.S. Bureau of Economic Analysis, Department of Commerce.
- Gelfand, Alan E. and Adrian F. M. Smith (1990), “Sampling-based approaches to calculating marginal densities”, *Journal of the American Statistical Association*, **85**, 398–409.
- Geman, D. and S. Geman (1984), “Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images”, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **6**, 721–741.
- Harvey, A.C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, Cambridge.
- Hecq, Alain and Jan P.A.M. Jacobs (2009), “On the VAR-VECM representation of real time data”, Technical report, Maastricht University.

- Jacobs, Jan P.A.M. and Jan-Egbert Sturm (2008), “The information content of KOF indicators on Swiss current account data revisions”, *Journal of Business Cycle Measurement and Analysis*, **4**, 163–183.
- Jacobs, Jan P.A.M and Simon van Norden (2011), “Modeling data revisions: Measurement error and dynamics of “true” values”, *Journal of Econometrics*, **161**, 101–109.
- Kim, Chang-Jin and Charles R. Nelson (1999), *State-Space Models with Regime Switching*, The MIT Press, Cambridge MA and London.
- Kishor, N.K. and E.F. Koenig (2012), “VAR estimation and forecasting when data are subject to revision”, *Journal of Business & Economic Statistics*, **30**, 181–190.
- Knetsch, T.A. and H.-E. Reimers (2009), “Dealing with benchmark revisions in real-time data: The case of German production and orders statistics”, *Oxford Bulletin of Economics and Statistics*, **71**, 209–235.
- Mankiw, N.G., D.E. Runkle, and M.D. Shapiro (1984), “Are preliminary announcements of the money stock rational forecasts?”, *Journal of Monetary Economics*, **14**, 15–27.
- Mankiw, N.G. and M.D. Shapiro (1986), “News or noise: An analysis of GNP revisions”, *Survey of Current Business*, **66**, 20–25.
- Patterson, K.D. (2003), “Exploiting information in vintages of time-series data”, *International Journal of Forecasting*, **19**, 177–197.
- Sargent, T.J. (1989), “Two models of measurements and the investment accelerator”, *The Journal of Political Economy*, **97**, 251–287.
- Schorfheide, Frank and Dongho Song (2014), “Real-time forecasting with a mixed-frequency VAR”, *Journal of Business & Economic Statistics* [forthcoming].
- Siklos, P.L. (2008), “What can we learn from comprehensive data revisions for forecasting inflation? Some U.S. evidence”, in D. Rapach and M.E. Wohar, editors,

*Forecasting in the Presence of Structural Breaks and Model Uncertainty*, Elsevier, Amsterdam.

Stone, Richard, D. G. Champernowne, and J. E. Meade (1942), “The precision of national income estimates”, *Review of Economic Studies*, **9**, 111–125.

Swiss National Bank (various issues), *Statistische Monatshefte*, Swiss National Bank, Zürich.

Zadrozny, Peter A. (2008), “Real-time state-space method for computing filtered estimates of future revisions of U.S. monthly chained CPI”, Mimeo, Bureau of Labor Statistics, Washington D.C.

Table A.1: Estimation results of AR models

	Indep. news-noise (AR): Spill		Corr. noise (AR): Spill		Corr. news (AR): Spill		Corr. news-noise (AR): Spill	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
$\theta_{1,11}$	0.758409098	1.037758739	0.839754388	0.127492028	0.955780712	0.153060661	1.038743622	0.1598582
$\theta_{1,12}$	3.10638E-07	1.03632E-05	2.15667E-07	1.01454E-05	-2.50639E-07	1.02762E-05	-8.46431E-08	9.71318E-06
$\theta_{1,21}$	4.04863E-07	1.00459E-05	-4.53689E-07	1.01855E-05	-2.54653E-07	1.01887E-05	7.43933E-08	9.82675E-06
$\theta_{1,22}$	0.561803813	0.114216385	0.596621271	0.10522417	0.762699826	0.077975533	0.763824617	0.089244972
$\theta_{2,11}$	7.35327E-07	9.7744E-06	-6.52005E-08	1.01321E-05	-2.11687E-07	9.75832E-06	-4.78149E-07	9.97428E-06
$\theta_{2,12}$	4.57072E-07	1.03894E-05	-3.61229E-07	9.64911E-06	-2.46467E-07	1.00577E-05	2.58333E-07	1.01574E-05
$\theta_{2,21}$	0.188196751	0.131912346	0.105400052	0.125842271	-0.015069113	0.130828883	-0.101925099	0.147577353
$\theta_{2,22}$	3.91559E-08	1.0103E-05	-7.03954E-07	9.79578E-06	6.24425E-07	9.69788E-06	-2.31651E-07	9.5882E-06
$T(3,3)$	-3.02176E-08	1.0175E-05	-6.33524E-07	1.03191E-05	-1.01902E-07	9.72076E-06	-4.51162E-07	1.02466E-05
$T(4,4)$	2.11645E-07	9.98082E-06	-1.59771E-08	9.8174E-06	1.25603E-07	9.96439E-06	3.86137E-07	9.8839E-06
$T(5,5)$	-7.41013E-08	9.92938E-06	2.71253E-08	9.57146E-06	1.44641E-07	9.71512E-06	1.66485E-08	1.02184E-05
$T(6,6)$	-4.0277E-07	1.02062E-05	3.82103E-07	9.93588E-06	1.71293E-07	1.02155E-05	-3.39913E-07	1.00824E-05
$T(7,7)$	-3.57651E-07	9.91675E-06	-1.68864E-07	1.01495E-05	-1.10803E-07	9.84127E-06	-6.6585E-09	1.00189E-05
$T(8,8)$	-1.01626E-07	1.0055E-05	-2.03128E-07	9.6206E-06	2.17302E-07	9.98927E-06	5.1981E-07	1.01772E-05
$T(9,9)$	3.81015E-07	1.00941E-05	-7.62731E-08	1.00146E-05	-4.91087E-07	1.00974E-05	2.40284E-07	1.01119E-05
$T(10,10)$	1.6334E-07	1.01389E-05	4.00007E-08	9.96448E-06	1.47472E-07	9.95957E-06	1.59967E-07	1.00549E-05
$T(11,11)$	5.14389E-07	1.01061E-05	-1.69487E-07	1.00849E-05	8.87801E-08	9.83268E-06	5.06557E-07	9.81982E-06
$T(12,12)$	9.27376E-09	9.40063E-06	1.43447E-07	9.64805E-06	1.94256E-07	9.84578E-06	-7.46396E-07	9.83115E-06
$T(13,13)$	4.72531E-08	1.04968E-05	-5.35508E-07	9.35376E-06	-2.19852E-07	9.88776E-06	-1.71897E-07	9.87926E-06
$T(14,14)$	-2.00089E-07	1.03417E-05	7.17818E-08	1.00468E-05	3.72608E-07	9.68739E-06	3.5559E-07	9.81346E-06
$\sigma_{xx}$	0.042993185	0.008342892	0.039400493	0.007470404	0.006861674	0.002188413	0.006802927	0.002274902
$\sigma_{xy}$	0	0	0	0	0	0	0	0
$\sigma_{yz}$	0	0	0	0	0	0	0	0
$\sigma_{yz}$	0.035823407	0.006705137	0.034260708	0.006489751	0.008013236	0.002562231	0.007827657	0.002415171
$\sigma_{xy_1}$	0.003098108	0.005479648	0.001019001	0.000986646	0.006128572	0.005208887	0.002026648	0.002286622
$\sigma_{xy_2}$	0.003632185	0.002379725	0.00189652	0.000911086	0.005299181	0.001440847	0.00239411	0.001957197
$\sigma_{xy_3}$	0.000731273	0.00029699	0.000732925	0.000328021	0.050258778	0.020701449	0.037425749	0.017066519
$\sigma_{xy_4}$	0.018753076	0.003652895	0.014480364	0.003221149	0.018715919	0.003649563	0.013256919	0.005118178
$\sigma_{xy_5}$	0.000833322	0.000799486	0.000830703	0.000439672	0.008854963	0.003079501	0.003108243	0.004156763
$\sigma_{xy_6}$	0.000704506	0.000288252	0.000724989	0.000298661	0.065681324	0.019085432	0.058482383	0.018531109
$\sigma_{x_5}$	0.013446639	0.006112991	0.016082413	0.003127075	0.010010291	0.005044925	0.015310956	0.003799056
$\sigma_{x_6}$	0.000722471	0.000296201	0.000830703	0.000365552	0.000695596	0.00025403	0.000778866	0.000286228
$\sigma_{x_8}$	0.002591923	0.002223697	0.004351678	0.001317469	0.00118293	0.00070952	0.004034678	0.002038332
$\sigma_{y_5}$	0.000852133	0.000471738	0.004376112	0.002165536	0.000852223	0.000529302	0.005958429	0.004406365
$\sigma_{y_6}$	0.00074088	0.000328323	0.000837084	0.000361938	0.00081667	0.000383207	0.000858976	0.000349532
$\sigma_{y_8}$	0.011445667	0.002444329	0.011662319	0.002324477	0.002919351	0.002106324	0.009299026	0.004576594

Table A.2: Estimation results of VAR models

	Indep. news-noise (VAR): Spill		Corr. noise (VAR): Spill		Corr. news (VAR): Spill		Corr. news-noise (VAR): Spill	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
$\theta_{1,11}$	0.542462142	0.194458063	0.48476308	0.220292156	0.503215765	0.202852402	0.475597268	0.18580483
$\theta_{1,12}$	0.284811653	0.216897857	0.141282523	0.231164432	0.233072703	0.225136221	0.208298803	0.208015639
$\theta_{1,21}$	-0.11302117	0.164446818	0.017204982	0.176393758	-0.125072811	0.17800026	-0.044552174	0.165447923
$\theta_{1,22}$	0.281644549	0.17991562	0.424535194	0.187675011	0.283794239	0.194990776	0.365910239	0.179834955
$\theta_{2,11}$	3.4284E-08	1.04368E-05	-5.94241E-08	9.727E-06	-2.42315E-07	1.04567E-05	-6.6694E-08	9.99474E-06
$\theta_{2,12}$	-4.16237E-07	9.96687E-06	3.54678E-07	9.7833E-06	-3.80822E-07	1.00705E-05	-5.40523E-08	9.87767E-06
$\theta_{2,21}$	-0.405526255	0.163664749	-0.395275965	0.179700606	-0.312188182	0.193848288	-0.381712494	0.172352758
$\theta_{2,22}$	-0.320056624	0.18331081	-0.371677568	0.196632493	-0.463597492	0.210574603	-0.546827446	0.193829505
$T(3, 3)$	5.18758E-08	1.00553E-05	-3.82012E-07	1.02839E-05	-1.3789E-07	1.00124E-05	2.64613E-07	9.97453E-06
$T(4, 4)$	3.59786E-07	1.02511E-05	6.47305E-08	1.0319E-05	5.8414E-08	9.98037E-06	2.92292E-08	1.03019E-05
$T(5, 5)$	5.08353E-07	1.01134E-05	3.58538E-07	9.49738E-06	2.94776E-08	9.92883E-06	1.556E-07	9.85092E-06
$T(6, 6)$	2.09303E-07	1.00381E-05	1.08195E-07	1.0232E-05	-1.0191E-07	9.86612E-06	5.90458E-07	9.98983E-06
$T(7, 7)$	2.6337E-07	1.01971E-05	8.64289E-08	9.86597E-06	-5.98495E-08	1.01747E-05	4.24173E-07	9.97464E-06
$T(8, 8)$	-3.10919E-07	9.41709E-06	-2.10652E-07	1.0189E-05	-2.73856E-07	9.7461E-06	4.14971E-08	1.00347E-05
$T(9, 9)$	-8.2651E-08	1.00581E-05	1.90416E-07	1.01719E-05	3.03619E-07	1.00168E-05	2.59102E-07	1.01845E-05
$T(10, 10)$	-1.70722E-07	1.01884E-05	1.89114E-07	1.01405E-05	-7.84577E-08	9.59116E-06	9.23441E-08	9.81912E-06
$T(11, 11)$	1.79118E-08	1.0291E-05	-1.53341E-07	1.05498E-05	6.27051E-07	1.02091E-05	-2.79435E-08	1.02575E-05
$T(12, 12)$	5.66422E-08	9.96058E-06	4.13665E-08	1.00362E-05	-3.79748E-07	9.82437E-06	-4.33618E-07	1.03385E-05
$T(13, 13)$	4.63186E-07	9.95329E-06	-3.18922E-07	1.07264E-05	-2.76846E-07	1.00071E-05	-1.6282E-07	1.01079E-05
$T(14, 14)$	2.6783E-07	9.91062E-06	4.64949E-07	1.00367E-05	-3.20849E-07	9.90237E-06	-2.47631E-07	1.00252E-05
$\sigma_{\pi\pi}$	0.029497531	0.00570427	0.025284861	0.005263996	0.031958222	0.005990255	0.02434002	0.004928423
$\sigma_{\pi\theta}$	0.029709897	0.006129542	0.022511849	0.005047358	0.030778141	0.006291025	0.023034104	0.005114635
$\sigma_{\theta\pi}$	0.029709897	0.006129542	0.022511849	0.005047358	0.030778141	0.006291025	0.023034104	0.005114635
$\sigma_{\theta\theta}$	0.039397197	0.007645488	0.030245978	0.005813965	0.038913053	0.007605303	0.0314488	0.006197337
$\sigma_{\pi\pi_1}$	0.015424851	0.002935685	0.003189267	0.004212507	0.016009143	0.003080065	0.006409874	0.002398049
$\sigma_{\pi\pi_2}$	0.000941077	0.000821349	0.001188606	0.000656428	0.004476126	0.00141649	0.003962434	0.00185747
$\sigma_{\pi\pi_3}$	0.000719636	0.000301712	0.000728355	0.000341386	0.000770706	0.000331335	0.000765296	0.000326858
$\sigma_{\pi\theta_1}$	0.018826973	0.003593663	0.012279697	0.00461057	0.018876971	0.003653538	0.004259336	0.002983383
$\sigma_{\pi\theta_2}$	0.010851539	0.00316482	0.00093901	0.000643629	0.011453263	0.002512208	0.009228987	0.004773202
$\sigma_{\pi\theta_3}$	0.000689876	0.00028866	0.00069994	0.000269115	0.000767238	0.000326155	0.000778024	0.000332544
$\sigma_{\pi\zeta_1}$	0.000769658	0.00034463	0.013823765	0.004689881	0.000760271	0.000334539	0.012545591	0.002875968
$\sigma_{\pi\zeta_2}$	0.000659538	0.000231976	0.000786821	0.000305098	0.000672182	0.000253828	0.000721056	0.000253167
$\sigma_{\pi\zeta_3}$	0.005548418	0.001539119	0.005221373	0.001374124	0.001757167	0.000949754	0.002441389	0.001876384
$\sigma_{\theta\zeta_1}$	0.000747069	0.000319737	0.0007008408	0.000361696	0.0007992	0.00035404	0.014470548	0.003390096
$\sigma_{\theta\zeta_2}$	0.00074269	0.000310377	0.000872099	0.000399707	0.000790467	0.000346778	0.000850326	0.000355316
$\sigma_{\theta\zeta_3}$	0.001321974	0.0002260264	0.011309575	0.002374137	0.001053353	0.000800452	0.003256624	0.004149875